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Electron Thermochemical Nonequilibrium Effects in Re-Entry Boundary Layers

DOYLE D. KNIGHT*
General Electric Company, Philadelphia, Pa.

Results of calculated nonequilibrium electron temperature and concentration characteristics in boundary layers are presented for re-entry conditions: U_∞ = 27,500–34,000 fps and altitude 110,000–150,000 ft. These results show the effects of reactions, collisions, electrical current, and electron reflection on the electron heat transfer and temperature. The electron conservation equations in boundary-layer form were solved in the quasi-neutral region with collisionless sheath boundary conditions or approximate boundary conditions derived from the collisionless sheath expressions and valid with collisions in the sheath. A numerical solution was obtained by a finite difference technique after a similarity transformation and quasi-linearization of the equations. The solutions show significant temperature nonequilibrium. Electron current and reflection, combined into a single parameter, reduces electron temperature and increases conduction of energy but does not explicitly affect electron concentration or diffusion of ionization energy.

Nomenclature

$\langle c_e \rangle$ = average electron speed = $(8kT_e/\pi m_e)^{1/2}$
c_{ve},c_{pe} = electron specific heats = $3k/2m_e$, $5k/2m_e$
D_A = ambipolar diffusion coefficient
e = magnitude of electronic charge
$\mathbf{E}$ = electric field
$\mathcal{E}$ = collisional energy transfer term, Eq. (5)
$F_{e,i}$ = electron, ion particle flux = $\frac{1}{4}n_e\langle c_e\rangle \exp(e\varphi/kTe)$, $\frac{1}{4}n_i\langle c_i\rangle$
I = ionization energy per unit electron mass
J_B = electrical current to body amps/cm²
k_f,k_r = forward and reverse reaction rates
K = thermal conductivity
L_e = electron Lewis number
m_s = species mass
n_s = species number density
$\dot{n}_e$ = electron source term due to reactions
p_s = species partial pressure
Pr = Prandtl number
$\mathbf{q}_e$ = electron heat flux vector
$Q_{C,D}$ = electron conduction and diffusion energy flux
$\hat{Q}_{C,D}$ = nondimensional energy flux
Q_{er}^* = averaged cross section = $\frac{1}{8}(\pi/2)^{1/2}(m_e/kT_e)^{3/2} \times (1/n_e)\int Q_{er}(c_e)c_e^3 f_e dc_e$
r_0 = radius of body from axis of symmetry
R_N = nose radius of body
Re = Reynold's number
T = temperature
u = x component of mass average velocity
U_∞ = flight velocity
$\mathbf{v}_s$ = species s mass average velocity
$\mathbf{v}^0$ = mass average velocity
$\mathbf{V}_s$ = species s diffusion velocity = $\mathbf{v}_s - \mathbf{v}^0$
x,y = distance parallel and perpendicular to body
α = degree of ionization = n_e/n_T
β = velocity gradient at stagnation point = $1/R_N(2p_F/\rho_F)^{1/2}$
γ = nondimensional degree of ionization = α/α_F
Γ = Damköhler number for homogeneous reactions
ζ = Damköhler number for heterogeneous reactions, Eq. (19)
η,ξ = transformed coordinates
θ = nondimensional electron temperature = T_e/T_F
Λ = current and reflection parameter, Eq. (22)
μ = viscosity
ρ = density
σ_{rs} = inelastic collisional energy transfer factor
τ_c = characteristic time for convection of a particle across boundary layer
τ_{coll} = characteristic time for collisional energy transfer, Eq. (10)
τ_r = characteristic reaction time, Eq. (12)
φ = potential difference across sheath
$\chi_{e,i}$ = fraction of electrons, ions impinging on body which are reflected
ψ = ratio of current to ion current

Subscripts

B = conditions at the body
e,i = electron, ion
F = conditions at boundary-layer edge
T = total
fr, eq = chemical reactions frozen, equilibrium

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* Aerodynamics Research Engineer, Space Sciences Laboratory; now Systems Engineer-Thermodynamics, Space Division. Associate Member AIAA.

Introduction

THE flow of ionized air over solid bodies has been a subject of considerable importance since the development of high-speed flight vehicles. Two aspects of the flow are generally of interest: heat transfer to the body and the electrical interaction between the body and charged particles in the flowfield.

Heat-transfer calculations for re-entry boundary layers including the effects of ionization[1,2] have been made with the assumption that all species are at the same temperature and frequently with the assumption of chemical equilibrium of all species. Jukes[3] illustrated that the electrons do not have to be at the same temperature as the heavy particles in boundary layers on cold bodies, and most of the work concerned with the electrical interaction in ionized boundary-layer flows[4–6] has considered electron thermal nonequilibrium. Chung[6] and Burke and Lam[5] treated the temperature as a dependent variable in their analyses for monatomic gases. Chung[7] later extended his analysis to air, including recombinations, but for conditions such that the flow was not in the boundary-layer regime but in a more rarefied viscous layer regime. These analyses were restricted to flows with less than 0.1% ionization, so that electron heat transfer would be small compared to the total.

Camac and Kemp[8] formulated an analysis of the heat transfer to a shock-tube end wall in argon, including electron chemical and thermal nonequilibrium. Numerical difficulties forced them to use a reaction rate 0.005 times the realistic value and to neglect collisional energy transfer. An interesting question that is the subject of this paper concerns the existence of electron thermal nonequilibrium in re-entry boundary layers with up to 5% ionization and the effect of electron thermal and chemical nonequilibrium on heat transfer.

The pertinent parameter in determining the extent of electron thermal nonequilibrium in a boundary layer was shown by Chung[6] to be the ratio of collisional energy transfer to conduction energy transfer for the electrons. Evaluating this parameter for the flight conditions given indicates that the electron temperature should be nonequilibrium, a conclusion which is questionable, due to the importance of inelastic collisions with molecules in air and the importance of electron-ion energy exchange with the higher degree of ionization. Chung's work[6] was also restricted to cases where electron-ion recombinations could be neglected; whereas in the present work they can not be.

The boundary layers in this work have electron concentrations high enough for electron-ion recombination by the three-body reaction

$$N^+ + 2e \underset{k_r}{\overset{k_f}{\rightleftarrows}} N + e$$

to be important. Characteristic flow times are two to three orders of magnitude larger than characteristic recombination times at the edge of the boundary layers of present concern. It should be noted that recombination is slowed by high electron temperature, and that electron temperature nonequilibrium is promoted by recombination since each three-body recombination results in the gain by an electron of the ionization energy. An interesting coupling, therefore, exists between electron thermal and chemical nonequilibrium.

This work establishes that electrons are present with temperature and concentration significantly above equilibrium levels in the stagnation point boundary layer on a cold body with nose radius 0.5–5.0 in at an altitude of 110,000–150,000 ft with stagnation temperature 9,000°K–11,000°K. Under these conditions, the degree of ionization in the shock layer is from 0.3% to 4.3%, with electron concentration 0.9×10^{16}–13.8×10^{16} electrons/cm^3. Electron temperature and heat transfer due to the electrons are studied to show the effects of collisions, reactions, electron current to the body, and electron reflection from the body.

Analysis

The analysis is based on species conservation equations for the electrons. Since this work is primarily concerned with electron phenomena, the neutral particle properties are assumed known, although, strictly speaking, they are weakly coupled to the electron properties at the ionization levels considered. Detailed neutral particle properties are necessary in evaluating electron reactions, collisional energy transfer, and transport coefficients. The conditions of this work are such that the inviscid shock layer is in equilibrium, and the neutral boundary layer may be assumed to be in equilibrium.

The electron equations studied are in boundary-layer form and restricted to the quasi-neutral region where the approximation $n_e = n_i$ may be made. The sheath is considered in order to derive boundary conditions for the quasi-neutral region and is assumed collisionless for this purpose. Approximate forms of the boundary conditions which are equally valid with collisions in the sheath are used when the collisionless assumption is invalid.

Conservation Equations

The characteristics of the electrons are governed by species continuity and energy equations in the following boundary layer form:

$$n_T \mathbf{v}^0 \cdot \nabla \alpha_e + (\partial/\partial y)(n_T \alpha_e V_e) = \dot{n}_e \tag{1}$$

$$c_{ve} m_e n_T \alpha_e (\mathbf{v}^0 \cdot \nabla T_e) + (\partial/\partial y) q_e = \mathcal{E} \tag{2}$$

The geometry and coordinates under consideration are standard in blunt body stagnation point work.[9] The dependent variable α_e is the degree of ionization n_e/n_T where n_T is the total number density. $\mathbf{V}_e$ is the electron diffusion velocity with respect to the mass average velocity $\mathbf{v}^0$.

Diffusion velocities in a two-temperature multicomponent mixture are[10]

$$n_T \alpha_e \mathbf{V}_e = -(p/kT_e) D_e \nabla[\alpha_e(T_e/T)] - n_T \alpha_e D_e (e\mathbf{E}/kT_e) \tag{3}$$

$$n_T \alpha_i \mathbf{V}_i = -n_T D_i \nabla \alpha_i + n_T \alpha_i D_i (e\mathbf{E}/kT) \tag{4}$$

where the diffusion coefficient D_e is given in terms of binary diffusion coefficients, D_{er}, by the relation $D_e = n_T / \sum_{r \neq e} n_r / D_{er}$ with an analogous relation for D_i.

Three body recombination of nitrogen is the only reaction of importance so $\dot{n}_e$ may be written $\dot{n}_e = k_r n_N n_e - k_f n_e^3$. The component of the heat flux vector normal to the wall q_e is the transport of random translational energy relative to the electron mass average velocity and as such is given by $q_e = -K_e(\partial T_e/\partial y)$.

$\mathcal{E}$ is a term representing energy transferred during collisions between electrons and other species. It is given here by the equation[10,11]

$$\mathcal{E} = \sum_r 4k n_e n_r \langle c_e \rangle Q_{er}^* \sigma_{er} (m_e/m_r)(T_r - T_e) - \dot{n}_e m_e I \tag{5}$$

which includes inelastic collisions by using an empirical factor σ_{er} with the standard[11] elastic collision energy transfer expression. The last term represents an electron energy loss in the amount of the ionization energy $m_e I$ during an ionizing collision with a nitrogen atom.

Quasi-Neutral Continuity Equation

The set of equations is incomplete because no equation is given for the electric field, $\mathbf{E}$, which appears in Eqs. (3) and (4). In the quasi-neutral region, where $n_e = n_i$, the electron continuity equation may be combined with an ion continuity equation to result in an equation which contains an "ambipolar" diffusion velocity expressible in terms of gradients as in Eq. (3) but without the explicit dependence on $\mathbf{E}$. Such an approach is useful only because ion and electron diffusion velocities can be neglected in the energy equation and the boundary conditions as discussed presently.

The steps[10] in combining ion and electron continuity equations are a generalization of previous work[12] to this case with reactions and with thermal nonequilibrium. The result is

$$n_T \mathbf{v}^0 \cdot \nabla \alpha + (\partial/\partial y)(n_T \alpha V) = \dot{n}_e \tag{6}$$

where the quasi-neutral assumption $\alpha_e = \alpha_i = \alpha$ has been used and D_i/T has been neglected compared to D_e/T_e due to the small mass of the electrons. The diffusion velocity appearing in this equation is a combination:

$$n_T \alpha V = n_T \alpha [V_i + (D_i T_e / D_e T) V_e] \tag{7}$$

which may be written, using Eqs. (3) and (4), as an ambipolar diffusion velocity,

$$n_T \alpha V = -n_T D_A (\partial \alpha / \partial y) - n_T D_i \alpha (\partial/\partial y)(T_e/T) \tag{8}$$

which depends on both the ion diffusion coefficient and the usual ambipolar diffusion coefficient $D_A = D_i(1 + T_e/T)$.

Discussion of Energy Equation

Order of magnitude analysis of the energy equation shows that a thermal boundary-layer analysis is valid if $Pr_e Re \rho_F/\rho_\infty \gg 1$, a condition which is met in this work due to the magnitude of $Re\rho_F/\rho_\infty$. The electron Prandtl number Pr_e is the ratio of convection to conduction of electron energy and is defined in terms of gas Prandtl number by $Pr_e = \frac{3}{5} Pr_g [\alpha (K_g/K_e)]_F$. For shock layer degree of ionization less than 0.1% this expression reduces to the form used by Chung[6] and has the constant magnitude 10^{-3}.

The source term $\mathcal{E}$ has two parts as in Eq. (5). The first in nondimensional form is

$$(R_N/U_\infty c_{ve} m_e) \sum_r 4k\sigma_{er}(m_e/m_r) n_r \langle c_e \rangle Q_{er}^* (T_r - T_e)/T_F \tag{9}$$

Since $R_N/U_\infty = \tau_c$ is a characteristic time for convection of particles across the boundary layer, this expression may be written as of order $o(\tau_c/\tau_{\text{coll}})$, where τ_{coll} is a characteristic time for collisional energy exchange evaluated at the boundary layer edge:

$$\tau_{\text{coll}} = \sum_r [(1/n_r \langle c_e \rangle Q_{er}^*) \tfrac{3}{8} (m_r/\sigma_{er} m_e)]_F \tag{10}$$

The second part of the source term, in nondimensional form, is

$$-(\dot{n}_e/n_{eF})(I/c_{ve}T_F) R_N/U_\infty \tag{11}$$

which may be written as of order $o[\Gamma(I/c_{ve}T_F)]$, where Γ is the Damköhler number for homogeneous reactions. It is the ratio of characteristic flow time τ_c to characteristic reaction time τ_r which is defined as

$$\tau_r = (1/k_f n_e^2)_F \tag{12}$$

An unique feature appearing with three body reactions is the competition between source terms—Eq. (9) an energy loss when $T_r - T_e$ is negative and Eq. (11) an energy gain when $\dot{n}_e$ is negative. Three-body reactions are effective in keeping the electrons hotter than the gas if the gain is larger, that is if

$$(c_{ve}T_F/I)\ \tau_r/\tau_{\text{coll}} < 1$$

For the present work this requirement is $\tau_r/\tau_{\text{coll}} < 11$.

The energy equation is not solved in nondimensional form and studied parametrically because the nondimensional parameters vary across the boundary layer. For example, Pr_e is constant only when the degree of ionization is less than 0.1% and changes by a factor of 20 across some of the boundary layers in the present work. Collisional energy transfer time, Eq. (10), does not adequately characterize collisional effects, since electron-ion collisions are most important at the boundary-layer edge but inelastic collisions with molecules dominate near the body.

The Lees-Dorodnitsyn transformation

$$\xi = \int_0^x \rho_F \mu_F u_F r_0^2 dx; \qquad \eta = \left(\frac{2\rho_F \beta}{\mu_F}\right)^{1/2} \int_0^y \frac{\rho}{\rho_F} dy$$

and a stream function $\partial f/\partial \eta = u(\eta)/u_F$ may be used to transform Eqs. (2) and (6) into ordinary differential equations in terms of η at the stagnation point. Defining new variables $\theta = T_e/T_F$ and $\gamma = \alpha/\alpha_F$, and using a prime to denote differentiation with respect to η, the result is

$$\left(\frac{n_T \rho D_A}{\rho_F \mu_F} \gamma'\right)' + \left[\frac{n_T \rho D_i}{\rho_F \mu_F} T_F \gamma \left(\frac{\theta}{T}\right)'\right]' + \frac{n_T f}{\rho} \gamma' = -\frac{\dot{n}_e}{2\rho\beta\alpha_F} \tag{13}$$

$$\left(\frac{\rho K_e}{\rho_F \mu_F} T_F \theta'\right)' + c_{ve} m_e \frac{k n_T f}{\rho} T_F \alpha_F \gamma \theta' = \frac{\mathcal{E}}{2\rho\beta} \tag{14}$$

Boundary Conditions

Equations (13) and (14) are the equations to be solved for the dependent variables θ and γ. The boundary conditions are derived by equating continuum expressions for particle and energy fluxes to expressions for fluxes across a sheath in which no collisions occur. Defining reflection parameters χ_e and χ_i as the fraction of electrons and ions, respectively, which are reflected back to the flow after striking the body, ion and electron fluxes across the sheath are

$$(1 - \chi_i)F_i = (1 - \chi_i)\tfrac{1}{4} n_i \langle c_i \rangle$$

$$(1 - \chi_e)F_e = (1 - \chi_e)\tfrac{1}{4} n_e \langle c_e \rangle \exp(e\varphi/kT_e)$$

evaluated at $y = \lambda$, a mean free path from the wall where $n_e = n_i$. The random ion flux $\langle c_i \rangle$ is calculated with heavy particle temperature at the sheath edge,[13] and φ is a negative quantity, the potential across the sheath. Jukes[3] discusses the energy flux across a collisionless sheath.

The continuum expression for particle flux in the quasi-neutral region, Eq. (8), is for a combination of electron and ion fluxes. Restricting this work to cases where $V_e = o(V_i)$ permits approximating this combination by the ion flux, since $D_i T_e / D_e T \ll 1$ in Eq. (7). The requirement of ion flux continuity is

$$(\partial \alpha/\partial y)_\lambda = [-(D_i/D_A)\alpha(\partial/\partial y)(T_e/T) + F_i(1 - \chi_i)/D_A]_\lambda \tag{15}$$

The requirement of electron energy flux continuity is

$$\left(\frac{\partial T_e}{\partial y}\right)_\lambda = \frac{F_e}{K_e}\left[\chi_e\left(2kT_e - \frac{3}{2}kT_B\right) + (1 - \chi_e)\left(|e\varphi| - \frac{1}{2}kT_e\right)\right]_\lambda \tag{16}$$

It is convenient to write the current as a multiple ψ of ion current, so that $\psi = 0$ for an insulated body. Thus, the electron flux is

$$F_e = F_i(1 + \psi)(1 - \chi_i)/(1 - \chi_e)$$

Because of the restriction to comparable ion and electron flux, ψ must be less than about 10 when electron current flows to the body. The temperature boundary condition is then written

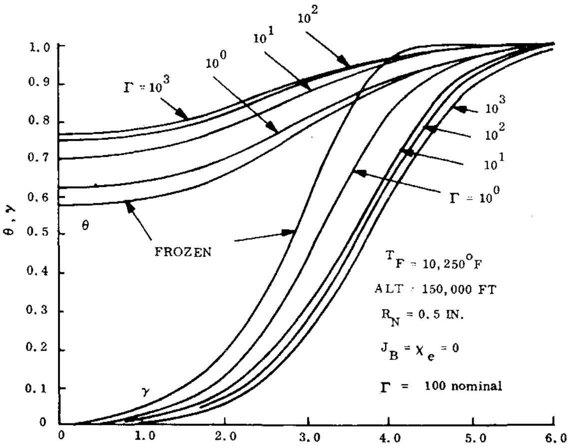

Fig. 1 Electron temperature and concentration boundary-layer profiles.

as

$$\left(\frac{\partial Te}{\partial y}\right)_\lambda = \frac{(1-\chi_i)(1+\psi)F_i}{K_e}\left[\frac{\chi_e}{1-\chi_e}\left(2kT_e - \frac{3}{2}kT_B\right) + \left(|e\varphi| - \frac{1}{2}kT_e\right)\right]_\lambda \quad (17)$$

In terms of transformed coordinates and nondimensional variables, the degree of ionization boundary condition [Eq. (15)] is

$$\frac{(1-\chi_i)\langle c_i\rangle_B\gamma_B}{4} = \rho_B\left(\frac{2\beta}{\rho_F\mu_F}\right)^{1/2}\left[D_A\gamma' - D_i\gamma\theta\frac{T_F}{T^2}T' + D_i\gamma\frac{T_F}{T}\theta'\right]_B \quad (18)$$

where the conditions at the body, denoted by B, are used instead of conditions at $y = \lambda$. Using $D_A = D_iT_e/T$ and a Damköhler number for heterogeneous reactions,[14]

$$\zeta = [(1-\chi_i)\langle c_i\rangle_B T_B/4D_i\rho_B\theta_B T_F](\rho_F\mu_F/2\beta)^{1/2} \quad (19)$$

the boundary condition becomes

$$[\zeta + (T'/T) - (\theta'/\theta)]_B\gamma_B = \gamma'_B \quad (20)$$

The gradient in electron temperature is frequently negligible in comparison to the heavy particle gradient, as will be shown presently. For a cold body $(T')_B$ may be approximated by T_F, so Eq. (20) contains the term T_F/T_B. Since $T_F/T_B > 1$, Eq. (20) can be approximated by $\gamma_B = 0$, even for small ζ in contrast to the noncatalytic ($\zeta = 0$) boundary condition ($\gamma'_B = 0$) in a one temperature analysis.

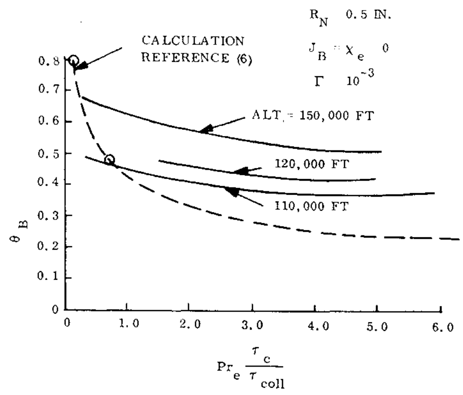

Fig. 2 Variation of electron temperature at the body with $Pr_e\tau_c/\tau_{coll}$.

The boundary condition on electron temperature [Eq. (17)] may be written using an electron Lewis number, $L_e = (\rho_e c_{pe} D_A/K_e)_B$, and the transformed coordinates and nondimensional variables:

$$\theta'_B = \frac{2}{5}\zeta L_e\left[(1+\psi)\left(\frac{|e\varphi|}{kT_F} - \frac{1}{2}\theta_B\right) + \frac{\chi_e}{1-\chi_e}(1+\psi)\left(2\theta_B - \frac{3}{2}T_B/T_F\right)\right] \quad (21)$$

The temperature boundary condition is affected by electron current and reflection in the same way, since they are combined into a bracketed expression in Eq. (21). This current and reflection parameter will be denoted by Λ,

$$\Lambda = (1+\psi)\left(\frac{|e\varphi|}{kT_F} - \frac{1}{2}\theta_B\right) + \frac{\chi_e}{1-\chi_e}(1+\psi)\left(2\theta_B - \frac{3}{2}T_B/T_F\right) \quad (22)$$

Since $L_e = 0(0.01)$ and ζ has values from 0.23 to 4.2 in the present work (with $\chi_i = 0$), the electron temperature gradient at the body will be negligible except in those cases with Λ large, i.e., χ_e and ψ greater than zero.

Increased electron current or reflection increases the electron temperature gradient and, therefore, causes the electron temperature to be closer to the heavy particle temperature. The resultant effects of temperature on the concentration boundary condition [Eq. (20)] are the only way in which current and reflection influence the electron concentration boundary condition in the quasi-neutral formulation.

For the flow conditions being considered, the boundary conditions may be approximated by

$$\gamma_B = 0, \quad \theta'_B = 0 \quad (23)$$

in all cases for which χ_e and ψ are zero. The main importance of the approximations is that they are valid with collisions in the sheath. The exact boundary conditions were used in all cases for which the sheath could be considered collisionless.

Solution

The equations to be solved are two-coupled nonlinear ordinary differential equations with split boundary conditions. Solutions were obtained by the technique of Ref. (15). Initial profiles were assumed in order to linearize the LHS of Eqs. (13) and (14), whereas quasi linearization was used for the source terms on the RHS. A finite difference numerical calculation and iteration scheme was programmed for the IBM 360 computer. Part of the iteration scheme[15] involved starting with the reaction rate much lower than its true value and progressively increasing it.

Quantities required in the solution include the reaction rate, electron thermal conductivity, inelastic collision factors σ_{er}, and electron and ion cross sections. The rate used is that recommended in Ref. (16) and recently confirmed[17] in the present electron temperature range. The uncertainty in the rate is approximately an order of magnitude.[18] Fay's[1] expression is used for the electron thermal conductivity. Inelastic collision factors were taken from Sutton and Sherman.[11] The electron cross sections were taken from Hochstim[19]; whereas ion cross sections were taken from Fay.[1] Neutral

Table 1 Conditions of calculations

Nose radius	R_N	0.5–50.0 in.
Altitude	Alt.	110,000–150,000 ft
Flight velocity	U_∞	27,500–34,000 fps
Degree of ionization	α_F	0.29%–4.3%
Stagnation temperature	T_F	9,000–11,000°K
Electron concentration	n_{eF}	0.88×10^{16}–13.8×10^{16} 1/cm^3

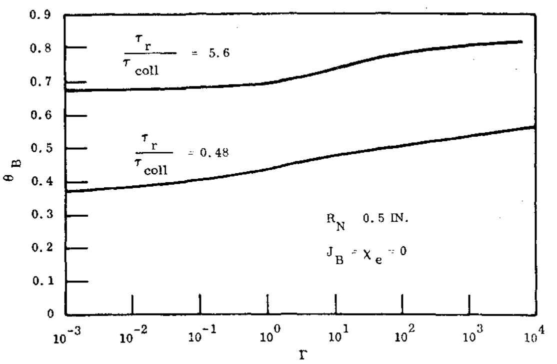

Fig. 3 Effect of recombination on electron temperature at the body.

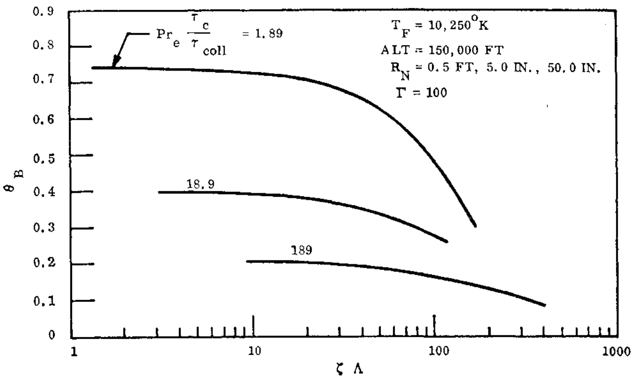

Fig. 4 Effect of electron current and reflection on electron temperature at the body.

particle concentrations, temperature, and velocity were taken from an equilibrium neutral boundary layer calculation.

Calculations and Results

Calculations were made for a number of specific cases with flow conditions corresponding to realistic flight or test conditions. Restrictions are imposed on the flow conditions covered by this work due to the assumptions made in the formulation, but the range of possible conditions produced a large variation in the important parameters. The flow conditions of the calculations are given and values are given for the important parameters; i.e., those already discussed qualitatively in regard to nonequilibrium electron temperature. The results are then presented and discussed in terms of these parameters.

The range of conditions for which calculations were made are given in Table 1 in terms of flight parameters and stagnation properties. The parameter ranges corresponding to Table 1 conditions are given in Table 2.

Figure 1 presents boundary-layer profiles of nondimensional electron temperature and nondimensional degree of ionization at the conclusion of several of the iterative steps. The nominal rate for this case gives $\Gamma = 100$. It may be seen from the figure that the electron temperature is significantly above the heavy particle temperature, which is about 3% of T_F at $\eta = 0$. Both electron temperature and concentration are nonequilibrium in the sense that increasing Γ changes them. The curve denoted "frozen" was calculated with no reactions ($\Gamma = 0$), and illustrates that with the parameter $Pr_e\tau_c/\tau_{coll} = 1.89$, the electron temperature decreases about 40% across the boundary layer. The effect of increased Γ is to raise θ since $\tau_r/\tau_{coll} = 1.97$, and three body recombinations are a source of energy comparable to collisional losses.

The degree of chemical nonequilibrium is indicated more dramatically by comparing the electron concentration obtained with the nominal rate, $\Gamma = 100$ and the conditions of Fig. 1 to the equilibrium electron concentration evaluated with the local electron temperature. The nonequilibrium concentration is two-orders of magnitude higher at $\eta = 1$. No electrons exist for equilibrium conditions near the body because no nitrogen atoms are present near the body, but the nonequilibrium concentration at the body is 3×10^{14} electrons/cm^3.

Table 2 Ranges of important parameters

τ_c	5.2×10^{-6}– 5.2×10^{-4} sec	τ_r/τ_{coll}	0.48–5.6
Pr_e	2.9×10^{-3}–17.2×10^{-3}	ζ	0.23–23
τ_r	3.5×10^{-9}–3.5×10^{-7} sec	ψ	0–10
τ_{coll}	0.72×10^{-8}–6.2×10^{-8} sec	χ_e	0–.99
Γ	15–10^4	J_B	0– −9.0 amps/cm^2
τ_c/coll	84–19,700	Λ	2–320
$Pr_e\tau_c/\tau_{coll}$	0.243–189	$\zeta\Lambda$	3.2–600

(τ_r, Γ based on nominal rate)

Figure 2 shows the effect of the parameter $Pr_e\tau_c/\tau_{coll}$ on the electron temperature at the body for the nearly frozen condition, $\Gamma = 10^{-3}$. The results are dependent on altitude because in these calculations n_{eF} was high enough for electron-ion collisions to dominate τ_{coll} which was evaluated at F, whereas toward the wall, collisions with neutral particles were the most important.

The values of θ_B may be compared to those of Chung[6] in which reactions were negligible. Smaller collision time near the body due to inelastic collisions in this work accounts for the electron temperatures being lower than those given by Chung for $Pr_e\tau_c/\tau_{coll} < 1$. Electron temperature values higher than Chung's for $Pr_e\tau_c/\tau_{coll} > 1$ may be due to the fact that in this range the Prandtl numbers of the present work were decreasing across the boundary layer.

The effect of reactions is to increase the electron temperature. Figure 3 shows the variation of θ_B with Γ for two values (0.48 and 5.6) of the ratio τ_r/τ_{coll}. As discussed previously, values less than about 11 indicate that energy gain due to recombinations is effective in keeping electrons hot. The difference in level of the curves is due to different $Pr_e\tau_c/\tau_{coll}$ values (0.24 and 5.4). Electron temperature increased with Γ at a slightly faster rate for the lower value of τ_r/τ_{coll} but the difference is not significant, so profiles of θ vs η were all spaced for different Γ values like the curves in Fig. 1.

The results presented thus far have illustrated the influence of energy transfer mechanisms in the boundary layer on the electron temperature. The temperature is also influenced by the conditions at the body via the boundary conditions. All the previous results were calculated under conditions where the approximate boundary conditions, $\gamma_B = 0$ and $\theta'_B = 0$, would be valid. In fact, the results for lower altitudes were calculated with these approximate boundary conditions but those for the 150,000-ft altitude were obtained with the collisionless sheath conditions. At 150,000 ft the collisionless assumption was reasonable, since the Debye length and mean free path were approximately equal at the sheath edge. The mean free path at lower altitudes was small enough to necessitate use of the approxima-

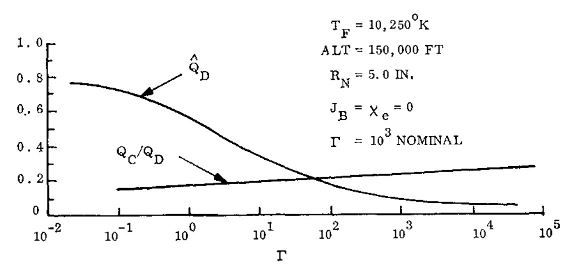

Fig. 5 Effect of recombination on electron energy transfer.

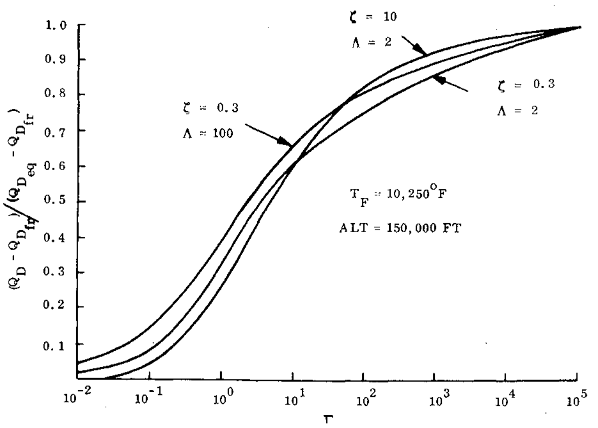

Fig. 6 Transition of Q_D from frozen to equilibrium.

tions. A calculation made with the approximations at 150,000 ft compared very well with the results obtained when the collisionless sheath conditions were used.

The effect of current and electron reflection is shown in Fig. 4 based on calculations with the collisionless sheath boundary conditions. The parameter $\zeta\Lambda$ has to be rather high for a reduction of θ_B because of the magnitude of the electron Lewis number in Eq. (21). Increasing body size not only increases ζ, Eq. (19), but also increases the parameter $Pr_e\tau_c/\tau_{coll}$, thus lowering the electron temperature level.

The results which have been shown in Figs. 1–4 correlate all the nonequilibrium electron temperatures of this study and show the effects of the phenomena—collisions, reactions, current, and reflection. The next series of figures show the effects of the various phenomena on heat transfer. These calculations were made with the collisionless sheath boundary conditions.

Figure 5 illustrates the effect of reactions on the nondimensional diffusion flux $\hat{Q}_D$ of ionization energy and shows the ratio of conduction flux Q_C to diffusion flux Q_D. These energy fluxes are defined as

$$Q_D = (1 - \chi_e)m_eI; \quad Q_C = -[K_e(\partial T_e/\partial y)]_B;$$
$$\hat{Q}_{D,C} = m_e[Q_{D,C}/I(n_TD_A)_F](\mu_F/2\rho_F\beta)^{1/2}$$

Both Q_D and Q_C decrease with large Γ due to decreased electron concentration near the body as reactions become more important. As Γ becomes large, $\hat{Q}_D$ levels off and approaches a constant value which may be called an "equilibrium" value. Note that the same type of asymptotic behavior is shown in the curves of θ and γ in Figs. 1 and 3. Figure 5 shows that even with Γ as large as the nominal value 10^3 of the present work, $\hat{Q}_D$ is still changing, and nonequilibrium effects on the electron heat transfer are significant.

The transition from frozen to equilibrium is shown in Fig. 6 to be independent of surface conditions which are contained by the ζ parameter in the range 0.3–10.0. The figure, in

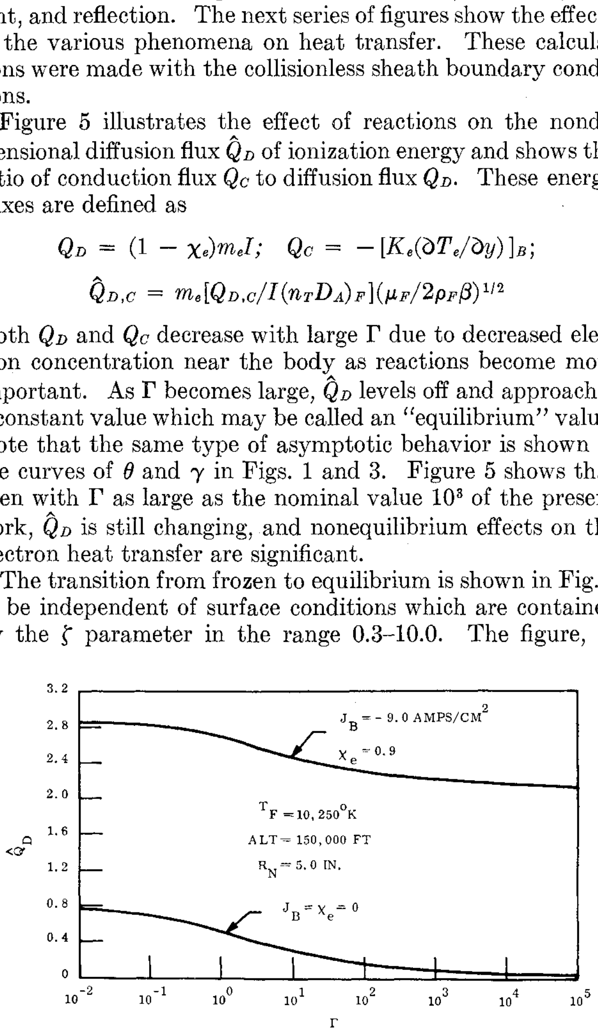

Fig. 7 Effect of electron current and reflection on $\hat{Q}_D$.

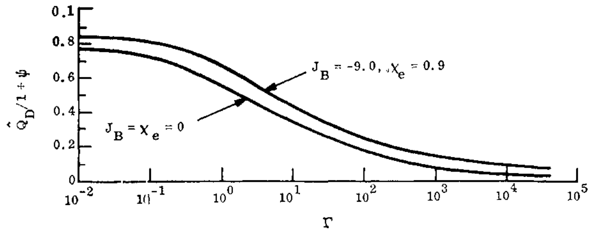

Fig. 8 Scaling of $\hat{Q}_D$ with ratio of electron current to ion current.

which equilibrium Q_D, denoted $Q_{D_{eq}}$, is defined to be the value with $\Gamma = 10^5$ shows that transition occurs between $\Gamma = 10^0$ and 10^3. A similar figure in Ref. 14, showing the same quantities as Fig. 6 but for a one-temperature mixture, indicated a dependence of transition on ζ. Decreasing ζ caused higher concentrations near the body which caused faster reactions and shifted the transition to the left along the Γ axis. In Ref. 14, a ζ value of 20 was equivalent to a completely catalytic body; whereas a ζ value of 0.1 was effectively noncatalytic. The transition for $\zeta = 0.1$ occurred almost three-orders of magnitude lower on the Γ scale than transition for $\zeta = 20$. No such shifting occurs in this work because as indicated previously, the boundary condition is approximately $\gamma_B = 0$, even with ζ small.

Figure 7 shows the effect of current and reflection on the heat transfer $\hat{Q}_D$. As Γ increased, the electron concentration at the body decreased so with J_B fixed the ratio of electron current to ion current increased. The ratio ψ went from 2.5 to 9 from left to right along the curve with $J_B = -9.0$ amps/cm².

Figure 8 shows the same data as Fig. 7, but with $\hat{Q}_D$ scaled by $1/(1 + \psi)$. The quantity $\hat{Q}_D/(1 + \psi)$ is almost independent of current and reflection, indicating that n_{eB} is affected little by current and reflection in the ranges considered in this work.

The influence of current and reflection on the ratio of conduction energy flux to diffusion energy flux is shown in Fig. 9. Conduction is more important in comparison to diffusion as Γ increases and as reflection increases. Figure 10 shows the correlation between conduction energy flux and the electron current and reflection. $\hat{Q}_C$ is seen to be almost linear with $\zeta\Lambda$.

Conclusions

The range of parameters covered by these calculations was broad enough to permit drawing conclusions concerning the effects of reactions, collisions, and electron current and reflection on nonequilibrium electron temperature and heat transfer. The electron temperature was higher than heavy particle temperature throughout the boundary layer for the conditions considered. The first significant conclusion from the results already presented is that even with Γ as large as 1000, electron chemical nonequilibrium was significant. Transition between an equilibrium type condition in which temperature and heat transfer did not change much as Γ was

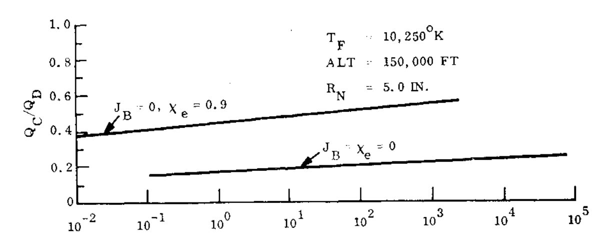

Fig. 9 Effect of reflection on Q_C/Q_D.

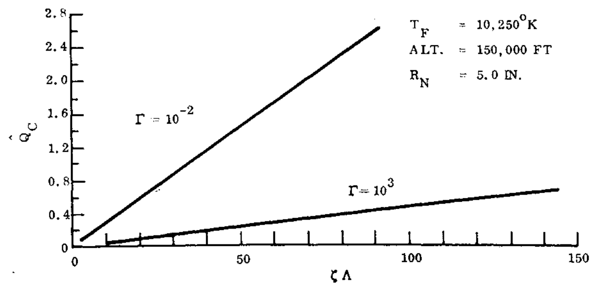

Fig. 10 Variation of conduction energy with electron current and reflection.

increased and a frozen condition occurred between about $\Gamma = 10^{\circ}$ and $\Gamma = 10^3$. Conduction and collisions had qualitatively the same effect on the temperature for the frozen case as shown by Chung.[6] Inelastic collisions near the body tended to cause electron temperatures lower than expected with Chung's results, but the variable Prandtl number at the higher degrees of ionization caused temperatures above Chung's results. The parameter, $Pr_e\tau_c/\tau_{coll}$ of Chung's work, was shown to be insufficient to completely correlate collisional effects on temperature because of the importance of electron-ion collisions at higher degrees of ionization and inelastic collisions in air. Electron reflection and current reduced the insulation effect of the sheath and resulted in a reduction in electron temperature in the boundary layer.

Reactions decreased the diffusion of ionization energy to the body and the conduction of energy to the body because of the reduction in electron number density at the body. Diffusion of ionization energy was insensitive to electron reflection but was in proportion to the ratio of electron current to ion current. Conduction of energy was increased by electron reflection; thus the total electron heat transfer was increased by reflection.

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